It is easily seen that when the same proportion of alcohol to water prevails, the contents of the alcohol in the first vessel will be$=a^2/(a+b)$.

$\therefore x$ must be$=\infty$.

II. Solution by G. B. M. ZERR. A. M., Ph. D.. Parsons, W. Va.

After the first operation, there are $a-c$ gallons of alcohol in the first vessel, and c gallons of alcohol in the second vessel. After the second operation, there are $a-2c+c^2(1/a+1/b)$ gallons of alcohol in the first vessel, and $2c-c^2(1/a+1/b)$ gallons of alcohol in the second vessel.

Let $A=c(1/a+1/b)$. Then, after the third operation, there are $a-3c+3Ac-A^2c$ gallons of alcohol in the first vessel, and $3c-3Ac+A^2c$ gallons of alcohol in the second vessel. After the nth operation there are

$$a-nc+\frac{n(n-1)}{2!}Ac-\frac{n(n-1)(n-2)}{3!}A^2c+\dots\pm A^{n-1}c=a+\frac{c(1-A)^n-c}{A}$$ gallons of alcohol, and $\dfrac{c-c(1-A)^n}{A}$ gallons of water in the first vessel, and

$$nc-\frac{n(n-1)}{2!}Ac+\frac{n(n-1)(n-2)}{3!}A^2c-\dots\pm A^{n-1}c=\frac{c-c(1-A)^n}{A}$$ gallons of alcohol, and $b+\dfrac{c(1-A)^n-c}{A}$ gallons of water in the second vessel.

$$\therefore \frac{Aa+c(1-A)^n-c}{c-c(1-A)^n}=\frac{c-c(1-A)^n}{Ab+c(1-A)^n-c}.$$

$$\therefore (1-A)^n=\frac{c(a+b)-Aab}{c(a-b)}=0. \quad \therefore n=-\infty, \text{ or } A=1.$$

$\therefore$ The result stated can only happen when $a=b=2c$, then $n=1$.

156. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

There exist no multiply perfect odd numbers of multiplicity n containing only n distinct primes.

Solution by JACOB WESTLUND, Ph. D., Purdue University, Lafayette, Ind.

If n denotes the multiplicity of a multiply perfect number $p_1^{a_1}p_2^{a_2}\dots p_i^{a_i}$, where p_1, p_2, ... are distinct primes, we have

$$n=\frac{p_1-\dfrac{1}{p_1^{a_1}}}{p_1-1}\cdot\frac{p_2-\dfrac{1}{p_2^{a_2}}}{p_2-1}\dots, \text{ and hence } n<\frac{p_1}{p_1-1}\cdot\frac{p_2}{p_2-1}\dots\frac{p_i}{p_i-1}.$$

Now, if $p_1>2$, we have

$$\frac{p_1}{p_1-1}\cdot\frac{p_2}{p_2-1}\ \cdots\cdots\ \frac{p_i}{p_i-1}\leqq\frac{3}{2}\cdot\frac{4}{3}\ \cdots\cdots\ \frac{i+2}{i}=\frac{i+2}{2}.$$

Hence we should have $n<\frac{i+2}{2}$, or $i>2n-1$.

PROBLEMS FOR SOLUTION.

ALGEBRA.

256. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Three men, A, B, and C, rented a pasture for a fixed amount, each to pay per month in proportion to the stock pastured. During the first month A put in 3 horses and B and C each some horses, and B paid for the month \$6, but A and C each defaulted payment. During the next month each put in one more horse, and C paid for the month \$7.20, but A and B each defaulted payment. During the next month each put in one more horse, and A paid his bill for the month, \$5, but B and C each defaulted.

Required: (1) the rent of the pasture per month; (2) the number of horses B and C each put in during the first month; and (3) the amount A, B, and C, each, owed for the unpaid service.

257. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

Solve (1) $x+y=10$, (2) $3x=\log_{10}y$.

258. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

Sum the infinite series $\frac{n^2}{(4n^2-1)^2}$ beginning with $n-1$, n being always odd.

CALCULUS.

216. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

Find the limit of the sum of the series

$$\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\cdots\cdots+\frac{n}{n^2+m^2},$$

when n and m are indefinitely increased. (Distinguish the several cases arising from the different *relative* values of m and n.)

217. Proposed by S. A. COREY, Hiteman, Iowa.

In *The Analyst*, Vol. II, p. 120, 1875, G. W. Hill finds by the method of mechanical quadrature the value of $\int_0^{\frac{1}{2}\pi}\frac{xdx}{\sin x[1+.16\cos^2 x]^{\frac{1}{2}}}$ to be 1.6576363.

Evaluate the definite integral by some other method and verify above result.